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Thoughts on John of Saxony's method for finding times of true syzygy

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Abstract

This article examines John of Saxony's iterative method for finding the times from mean to true syzygy (i.e., conjunction or opposition of the Moon and Sun). It argues that the method, composed c. 1330, contains several ambiguities, but is so robust that only one of these ambiguities affects the time correction. Furthermore, the method yields times of true syzygy that correspond, to the nearest minute, to the time when the true elongation, as computed by the planetary equations of the 1483 Alfonsine Tables, makes its closest approach to 0° or 180°. Hence John's method yields "exact" Alfonsine times, unlike all other known medieval methods or tables that only approximate those results. It will also be shown that John Somer (1380s) and Regiomontanus (1440–1450s) wielded John's method with considerable computational skill.

Zusammenfassung

Die iterative Methode des Johannes de Saxonia die Zeiten von mittleren bis zur wahren Opposition bzw. Konjunktion der Sonne und Mond finden zu können wird untersucht. Obwohl die Methode, zusammengefasst um 1330, einige Zweideutigkeiten innehatte, sind dieselben meistens durch die Stärke der Methode ausgeglichen. Die Ergebnisse der Methode stimmen bis zur Minutengenauigkeit mit den Ergebnissen der Planetengleichungen der 1483 Alfonsinischen Tafeln überein. D.h., die Methode Johannes gibt "exakten" alfonsinischen Zeiten der Oppositionen bzw. Konjunktionen, im Gegensatz zu allen anderen mittelalterlichen Methoden oder Tafeln, die nur annährend diese Zeiten ergeben. Es wird auch gezeigt, dass John of Somer (1380–1390) sowie Regiomontan (1440–1450) die Methode Johannes geschickt ausgeübt haben.

Résumé

Cet article examine la méthode itérative de Jean de Saxe pour déterminer le temps des conjonctions et oppositions vraies entre la Lune et le Soleil. Bien que cette méthode, composée vers 1330, contienne trois ambiguïtés, elle est si puissante qu'une seule d'elles a un effet sur la correction du temps. En outre, elle produit les temps de la conjonction ou opposition vraie qui, à une minute près, correspondent au moment où l'élongation vraie, calculée à l'aide des

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équations planétaires alphonsines de 1483, s'approche le plus de 0° ou 180°. Par conséquent, la méthode de Jean de Saxe produit des temps alphonsins "exacts," un résultat que toutes les autres méthodes et tables médiévales n'obtiennent qu'approximativement. On démontrera que John Somer (dans les années 1380) et Regiomontanus (dans les années 1440–1450) ont utilisé la méthode de Jean de Saxe avec une habileté calculatrice considérable.

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Judging from extant calendrical and astrological computations, it would appear as if users of late medieval Latin and Hebrew astronomical tables concerned themselves more with new and full moons (syzygies) and eclipses than with any other phenomena of planetary motion. Literally dozens of manuscripts from the 14th and 15th centuries contain lists of dates and times for all the syzygies and eclipses over one or more Metonic cycles, as do nearly all of the incunable printed ephemerides and calendars. For example, the calendars of John Somer and Nicholas of Lynn, astronomers both linked to Oxford, provide such data for the years from 1387 to 1462. In 1448 the young Regiomontanus began computing annual ephemerides, including daily longitudes for all the planets, and drawing up horoscopes for the times of each syzygy. His extant notebooks contain such horoscopes for nearly every year through 1464, often listing only the times of syzygy and not the planetary positions [Eisner, 1980; Mooney, 1998; Vienna ÖNB, cvp 4988]. Regiomontanus's massive ephemerides, printed ca. 1474 (nine further editions to 1494), include syzygy times (as well as daily planetary longitudes) from 1475 to 1504; his smaller calendar, which he printed in Latin and German editions, also ca. 1474 (thirteen further editions to 1496), presents only syzygy and eclipse times for three cycles from 1475 to 1531 [Zinner, 1937]. A calendar by Jakob Pflaum, appearing in two German editions and one Latin edition in 1477 and 1478, offers syzygy and eclipse times for 1477–1552. Bernart of Grannollach's *Lunaria*, appearing between 1485 and 1525 in at least 60 editions printed in Spain, in France, and mostly in Italy, lists syzygy and eclipse times from 1485 to 1550 [Chabás and Roca, 1998, ISTC]. The most popular genre of printed astronomical texts during the incunabula period, the annual broadside almanacs and multileafed practica or judica, invariably include syzygy and eclipse times and only rarely mention other quantitative planetary data.[1] In these various sources, the times of syzygy and eclipses almost without fail are stated to the nearest minute. Computing syzygies to a precision of minutes might well have been the most common preoccupation of late medieval astronomers.[2]

[1] The roughly 30,000 incunable editions printed across Europe include nearly 500 editions of almanacs and 400 editions of practica. See ISTC.

[2] A collection of astrological and calendrical German texts, compiled c. 1400 and called the *Volkskalender* by E. Zinner, often is accompanied in the manuscripts by lists of syzygy times, specified to a precision of the nearest minute, for one or more Metonic cycles. Yet all of the syzygy lists I have seen accompanying both German and Latin versions of the *Volkskalender* present times of mean syzygy (i.e., equal intervals between the successive syzygies), and thus are not relevant to this discussion of procedures for computing times of true syzygy. See Wolfenbüttel, HAB 81.24.Aug.fol; Augsburg, Universitätsbibliothek III, 1.4°1, ff. 31v–33v, 122r–25v, 2*r–7*v; Munich, BSB Cgm 397, ff. 1r–6v, Clm 5595, ff. 37r–43v, 100r–5v, Clm 5879, ff. 212r–18v. Printed editions of the *Volkskalender* began to appear in the 1480s (see Zinner [1952], Amelung [1978], Keil [1983], Brévart [1988]). John of Gmunden's calendar, widely extant in many manuscripts, also lists times of mean syzygies, for cycles

As is well known, Ptolemy reduced his planetary models to computational algorithms that are functions of time. One enters a time and computes a position in longitude or latitude. The algorithms do not operate in reverse; one cannot enter a position and compute a time. For synodic phenomena such as oppositions, conjunctions, eclipses, and stationary points, the *Almagest* in Books VI and XII presents techniques for approximating times by the use of velocities. For the phenomena of interest here, the times of true syzygy, the approximations are complicated by the fact that the Moon's rapidly changing velocity cannot be considered constant over the time interval from mean to true syzygy. To solve this problem, Ptolemy proposed an iterative process, in which the time of true syzygy is estimated by assuming a constant lunar velocity, the true elongation is computed for that time, and if the latter is not equal to 0° or 180°, a new time of true syzygy is estimated and the process is repeated. Later medieval astronomers, as J. Chabás and B.R. Goldstein have shown, reduced the problem to a single step by means of single- or double-entry tables, themselves computed from velocity tables for the Sun and Moon. Both techniques—the iterative and tabular—are approximative; in the hands of medieval users, both were wielded to produce results to a precision of minutes of time.

As noted by Chabás and Goldstein, astronomers working within the tabular tradition formulated several different approximative solutions in tables with various arguments. Ptolemy's iterative approach, on the other hand, passed essentially unchanged into the work of al-Battani, the canons circulating with the tables of al-Khwarizmi, the Toledan Tables, and the Castilian canons to the Alfonsine Tables. Its only significant elaboration came at the hands of the Parisian, John of Saxony, in the canons he composed c. 1330 for the Alfonsine Tables. These canons appear in many manuscript versions of the Alfonsine Tables as well as in their *editio princeps*, printed in 1483 by Ratdolt in Venice. As described by Chabás and Goldstein, John of Saxony:

> ... offered a more sophisticated solution using Ptolemy's lunar models. He made allowances for the variation in the lunar velocity in the time interval between mean and true syzygy, and introduced a method of successive approximations of Δt, first to the nearest hour, and then to the nearest minute of hour. This yields an improvement in accuracy, but involves a lot of computation that most practitioners in the late Middle Ages were probably not prepared to follow [Chabás and Goldstein, 1997, 97].

In the commentary to his edition of John of Saxony's canon, E. Poulle summarized the method and demonstrated its use by computing step by step the time of true syzygy for 19 July 1327, a date used by John for examples in his canon. Chabás and Goldstein also outlined John's procedure and worked the example for July of 1327 [Poulle, 1984, 208–219; Chabás and Goldstein, 1992, 269–271].

These earlier analyses have noted several ambiguities in John's canon. Yet by considering only the July 1327 example, the studies by Poulle and Chabás and Goldstein do not expose what might be called the "robustness" of John's procedure. That is, what appear to be ambiguities in John's canon may be insignificant since they do not affect the outcomes of that procedure, i.e., the time correction, Δt, from mean to true syzygy, specified to a precision of the nearest minute. In particular, it will be shown that John's method yields identical results, regardless of which of the available lunar velocity tables are used. Furthermore, the earlier analyses have not indicated that John's procedure, unlike the tabular techniques, yields times of true syzygy that correspond, to the nearest minute, to those times when the true elongation, as computed by the planetary equations of the 1483 Alfonsine Tables, makes its closest approach to 0°or 180°. Of all the medieval procedures for approximating times of true syzygy, John's most closely

from 1416 to 1434, and 1420 to 1496. I have consulted copies of Gmunden's calendar in Munich, BSB Clm 5595, 37r–43v, Cgm 303, 3r–21r; London, BL Add 24070, 2r–4v; and Vienna, ÖNB cvp 2440, ff. 1v–12v (see Zinner [1925, #3606–3687]).

reproduces the results of the time-dependent algorithms of the 1483 Alfonsine Tables. At a precision of minutes, John of Saxony's approximations become exact.

Three ambiguities in Chapter 22 of John's canon

As explicated by Chabás and Goldstein (using their notation), John's method breaks the time correction Δt into two components, to be computed separately:

$$\Delta t = \tau + \tau^*. \tag{1}$$

For the first term,

$$\tau = \frac{-\eta}{\upsilon_m(\alpha) - \upsilon_s(\kappa) + \delta}, \tag{2}$$

where η is the true elongation at the time t of mean syzygy (for consistency of algebraic signs, $\eta = \lambda_m - \lambda_s$), $\upsilon_m(\alpha)$ and $\upsilon_s(\kappa)$ are the hourly lunar and solar velocities in anomaly, respectively, at time t, and δ is a small empirical correction factor, never larger than about 8 arcsec. For the lunar velocity term, John defines the "equated lunar argument" (line 34) as[3]

$$\alpha = \bar{\alpha} - \frac{13}{24}\eta, \tag{3}$$

where $\bar{\alpha}$ is the mean lunar anomaly at time t. For the solar velocity term, John simply writes that the "solar argument" is to be used (line 42). Chabás and Goldstein assume that the "true solar argument" is meant, as I have written in Eq. (2). In his worked example, however, Poulle employs the mean solar anomaly as argument. John defines the correction term as follows, with η given in degrees:

$$\delta = \pm 0;01 \bullet \big(\mathrm{Int}(|\eta|) - 1\big).^4 \tag{4}$$

As described in the canon, lines 59–62, $\delta \geqslant 0°$ for $\alpha \geqslant 180°$, and vice versa. For the second term, John computes the true elongation at $t + \tau$, and again at $t + \tau + 0;01$ day (i.e., 24 min), and defines

$$\tau^* = \frac{-\eta^*}{d\eta}, \tag{5}$$

where η^* is the true elongation at $t + \tau$ and $d\eta$ is the change of true elongation over the 24 minutes.[5]

John's description of these two steps appears to contain several ambiguities, all of which arise in Eq. (2) and Eq. (5). The first of these concerns which of Ptolemy's lunar models is intended to be employed for computation of true elongations [Petersen, 1969; Neugebauer, 1975, 68–98]. For the first step at the time of mean syzygy, John instructs users to "enter the table of the lunar equation with the mean lunar argument, and take the equation so that you can find the true longitude of the Moon" (lines 13–15). With no reference to the lunar equation of center or proportional parts, John here appears to be describing Ptolemy's initial lunar model. Yet since at mean syzygy the lunar equation of center is 0°,

[3] Poulle [1984, 81], translates argumentum lune equatum as "argument vrai de la lune."

[4] To match John's canon, lines 55–62, the true elongation must be converted to absolute value before being made into an integer.

[5] For oppositions, 180° must be removed from η and η^* in Eqs. (2) through (5). To match John's canon, my equation (5) revises Chabás and Goldstein's equation (7) by making the sign of the numerator negative.

Ptolemy's initial and final lunar models are equivalent at this time, and the choice of a lunar model becomes irrelevant in step one.

For step two, however, the true elongation must be computed for times other than that of mean syzygy, i.e., for times when Ptolemy's initial and final lunar models are not necessarily equivalent. Here, the canon instructs users merely to "calculate precisely" the true lunar longitude (lines 90–91) without specifying the lunar model to be employed. Chabás and Goldstein suggested, without argument, that Ptolemy's final lunar model is to be used. Poulle did not discuss this question, but in his worked example for 1327 he sought the proportional minutes at $t + \tau$, something required only by the final lunar model [Chabás and Goldstein, 1992, 270; Poulle, 1984, 217]. Interestingly, for the 1327 example, Poulle found that the proportional minutes at $t + \tau$ are zero. It can easily be shown that in many cases after John of Saxony's first step, the proportional minutes will be zero. According to the lunar equations of the 1483 Alfonsine Tables, the proportional minutes remain zero as long as $2\bar{\eta} < 12°$. Since the 1483 Alfonsine mean elongation moves at a rate of slightly more than $12°/\text{day}$, $2\bar{\eta}$ requires about 12 h to move 12°. Thus, whenever the absolute value of the time from mean to true syzygy exceeds 12 h, the proportional minutes become 1 rather than 0, and slight differences begin to emerge between true longitudes computed by Ptolemy's initial and final lunar models. The 1327 example worked by Poulle and Chabás and Goldstein is one for which the proportional minutes remain zero. A wider range of examples, to be discussed below, will reveal that this ambiguity over Ptolemy's lunar models in the second step of John's procedure can introduce deviations of up to 5 min of time in Δt, but never in more than about half of the syzygies for a given year. Occasionally (e.g., in 1380) spacing of the syzygies is such that $2\bar{\eta}$ remains $<12°$ for every conjunction throughout a year. Nonetheless, this ambiguity in lunar models to be used at time $t + \tau$ can introduce uncertainties in Δt that exceed 1 min of time.

A second ambiguity arises when John's canon does not specify whether the solar velocity term in Eq. (2) is a function of the mean or true solar anomaly. The solar equation of the 1483 Alfonsine Tables reaches a maximum of 2;10°; an ambiguity over whether to use the mean or true solar anomaly would at most shift the argument for entering a solar velocity table by 2;10°. Given the slow rate of change of the solar velocity, this would at most shift the solar velocity term by 1 arcsec/h. As we shall see below, such a small variation in the denominator of Eq. (2) would not in general affect the final time correction being determined by Eq. (1) to a precision of the nearest minute.

A further and potentially serious ambiguity arises in Eq. (2), for the canon does not specify which table of solar and lunar velocities is to be used. John wrote (lines 36–42): "Note that there exists a very good table, in my opinion, in which one easily finds the hourly movement of the Moon and Sun, and as many tables have been constructed for this end, the most exact that I have seen is by John of Lignères, in which one enters with the solar argument and the lunar argument." Unfortunately, it is impossible to identify the "most exact" table to which the canon refers. In his commentary, Poulle observed that 14th-century manuscripts contain many velocity tables, "astonishingly variable" in content, that have been or might be attributed to John of Lignères (fl. first half of 14th century). Other studies by Goldstein and Porres also have found considerable variety in medieval Latin and Arabic tables of lunar velocity, as can be seen in Table 1. Similar variety appears in the accompanying solar velocity tables.

Another use of velocity tables in John's method might arise in Eq. (5), which as Chabás and Goldstein have noted can be written

$$\tau^* = \frac{-\eta^*}{d\lambda_m - d\lambda_s}, \tag{6}$$

Table 1
Medieval lunar velocity tables

Author	Minimum	Maximum	Source
John of Lignères (1)	0;29,00°/h	0;37,56°/h	Porres [forthcoming]
Levi ben Gerson (2)	0;29,35	0;36,56	Goldstein [1974, 182]
John of Lignères (2)	0;29,37	0;36,53	Goldstein [1992, 12–13]
John of Genoa, John of Lignères (3)	0;29,37,13	0;36,58,54[a]	Goldstein [1992, 12–13], Porres [forthcoming]
John of Montfort	0;29,38	0;36,52	Goldstein [1996, 190]
Al-Khwarizmi	0;30,12	0;35,40	Goldstein [1996, 190]
Al-Battani, Toledan Tables, Levi ben Gerson (1), John of Lignères (4)	0;30,18	0;36;04	Nallino [1899–1907, 2:88], Goldstein [1974, 182], Porres [forthcoming]
Ibn al-Raqqam	0;30,21	0;36,01	Goldstein [1996, 190]
1483 Alfonsine Tables	0;30,21	0;36,25	Alfonso [1483]

[a] Goldstein [1992, 12–14] has shown that the maximum value for this table should be 0;36,53,20°/h, according to the algorithm that produces the other entries in the table.

where the velocity terms might be found by entering tables for lunar and solar velocities for a "minute of a day" (i.e., 24 minutes) at time $t + \tau$. The 1483 Alfonsine Tables, for example, include two sets of velocity tables, for hours and "minutes of a day." Use of such "minutes of a day" table would eliminate the need to compute the true elongation at time $t + \tau + 0;01$ day from the lunar equations. However, John's canon clearly describes the computation of the true longitudes at $t + \tau$ and $t + \tau + 0;01$ day, not mentioning velocity tables as he did when describing Eq. (2), so this potential procedural ambiguity, i.e., whether to use velocity tables for minutes of a day or compute true elongations directly, cannot be ascribed to the canon.

The robustness of John's method

The significance of these ambiguities in John of Saxony's canon can best be explored by computing examples with the above equations. Only by entering the algorithms with differing values for the mean solar and lunar anomalies can the robustness of John's method, i.e., its imperviousness to the ambiguities in its description, be recognized. In all of the following computations, I have used entries for equations and velocities as they appear in the manuscripts (edited as in Table 1) or the 1483 Alfonsine Tables, and have interpolated linearly. Intermediate steps are not rounded unless otherwise noted.

Table 2 presents the results of using several different lunar velocity tables in Eq. (2), assuming that the remaining computational steps remain unchanged. For reasons that will become apparent shortly, Table 2 includes all conjunctions occurring over the two-year period from 1387 to 1388. The first three columns list dates and mean values for the successive mean conjunctions, as derived from the 1483 Alfonsine Tables. Values for the mean solar and lunar anomalies in cols. 2–3 have been rounded to degrees; for the computations, these values were not rounded. For the computation of true elongations at times of mean syzygy $t, t + \tau$, and $t + \tau + 0;01$ day, I use the solar and lunar equations of the 1483 Alfonsine Tables and Ptolemy's final lunar model. The solar velocity, consistently taken from the 1483 Alfonsine Tables,

Table 2
Comparing velocity tables in Eq. (2) for conjunctions of 1387–1388

Mean conjunction			1483 Alf. Tables			John of L. 1		John of L. 2		John of L. 4	
1	2	3	4	5	6	7	8	9	10	11	12
Date	$\bar{\kappa}$	$\bar{\alpha}$	τ	τ^*	Δt	τ	τ^*	τ	τ^*	τ	τ^*
	°	°	h	m	h	h	m	h	m	h	m
20.Jan	217	144	8;13	−11	8;02	7;52	10	8;02	0	8;12	−10
18.Feb	246	169	5;09	6	5;15	4;56	19	5;04	11	5;11	4
19.Mar	276	195	1;18	−2	1;16	1;14	2	1;16	0	1;18	−2
18.Apr	305	221	−3;09	4	−3;05	−3;02	−3	−3;05	0	−3;09	4
17.May	334	247	−7;08	4	−7;04	−7;02	−2	−7;04	0	−7;08	4
15.Jun	3	273	−9;51	−2	−9;53	−9;57	4	−9;53	0	−9;52	−1
15.Jul	32	299	−10;48	−8	−10;56	**−11;04**	**7**	−10;55	−1	−10;48	−8
13.Aug	61	324	−9;37	−12	−9;49	−10;01	12	−9;48	−1	−9;37	−12
12.Sep	90	350	−6;19	−10	−6;29	−6;38	9	−6;29	0	−6;19	−10
12.Oct	119	16	−1;27	−2	−1;29	−1;31	2	−1;29	0	−1;27	−2
11.Nov	148	42	4;03	4	4;07	4;12	−5	4;07	0	4;03	4
10.Dec	178	68	8;51	4	8;55	9;02	−7	8;56	−1	8;51	4
09.Jan	207	93	11;41	2	11;43	**11;45**	**−1**	11;39	4	11;42	1
08.Feb	236	119	11;55	−6	11;49	11;40	9	11;46	3	11;55	−6
08.Mar	265	145	9;30	−14	9;16	9;05	11	9;16	0	9;29	−13
06.Apr	294	171	5;00	−6	4;54	4;46	8	4;54	0	5;02	−8
06.May	323	197	−0;30	1	−0;29	−0;28	−1	−0;29	0	−0;30	1
04.Jun	352	223	−5;59	6	−5;53	−5;46	−7	−5;52	−1	−5;59	6
03.Jul	21	248	−10;22	4	−10;18	−10;15	−3	−10;17	−1	−10;22	4
02.Aug	50	274	−12;51	−8	−12;59	−12;59	0	−12;53	−6	−12;52	−7
31.Aug	79	300	−12;44	−16	−13;00	−13;04	4	−12;53	−7	−12;44	−16
30.Sep	109	326	−9;54	−13	−10;07	−10;19	12	−10;06	−1	−9;54	−13
30.Oct	138	352	−4;40	−8	−4;48	−4;54	6	−4;47	−1	−4;40	−8
28.Nov	167	17	1;49	2	1;51	1;54	−3	1;51	0	1;49	2
28.Dec	196	43	8;01	8	8;09	8;19	−10	8;09	0	8;01	8

is assumed to be a function of the true solar anomaly, as written in Eq. (2). Sources for the four lunar velocity tables here compared are identified in Table 1.

The four lunar velocity tables here compared reflect the maximal divergences among the known (i.e., edited) medieval velocity tables listed in Table 1. Yet when used in John's equation (2), each of these four velocity tables yields identical results for the total time correction ($\tau + \tau^*$) for the 25 conjunctions of 1387–1388, with only two exceptions (bold font). In those two cases, the lunar velocity table of John of Lignères 1 produces a time correction that varies by 1 min from the corrections of the other three lunar velocity tables. Thus, the second step of John's method, in which τ^* is computed directly from the true elongations at time $t + \tau$ and $t + \tau + 0;01$ day, is powerful enough to overcome slight variations in the value of τ that result from employing differing lunar velocity tables in Eq. (2). Indeed, so powerful is Eq. (5) that one can compute identical time corrections for each of the 1387–1388 conjunctions simply by replacing the velocity tables in Eq. (2) with the difference of the mean velocities, as specified in the 1483 Alfonsine Tables (12;11,26,42 °/day), and dropping the delta correction.[6] That is, Eq. (2) can replaced

[6] For the conjunctions of 1387–1388, Eq. (6) yields values for τ^* that range from −58 to 42 min.

by the following, with no change in the total corrections computed to the nearest minute of time for the 1387–1388 conjunctions:

$$\tau = \frac{-\eta}{\bar{\upsilon}_m - \bar{\upsilon}_s}. \tag{7}$$

Interestingly, the comparisons of Table 2 also reveal that the lunar velocity tables of John of Lignères 2 appear to minimize the values of the τ^* correction. For half of the cases in cols. 9–10, the first-step correction already yields a true elongation of 0°. This version of John of Lignères's velocity tables, along with other very similar tables that have been attributed to John Montfort and John of Genoa, belongs to a large and still rather unexamined set of manuscripts that compose what J. Chabás and B.R. Goldstein have called the "corpus of Alfonsine Tables" [Chabás and Goldstein, 1992, 280; Goldstein, 1992, 11–13]. The comparisons of Table 2 suggest that John of Saxony may well have tailored the structure of Eq. (2), with the apparently empirical delta function, to fit this set of velocity tables within the "Alfonsine corpus." Such a conclusion would also make understandable John's canon, lines (89–93), where he notes that a true elongation of 0° or 180° can sometimes be attained after only the first step, i.e., Eq. (2), of his method. For nearly half of the conjunctions of 1387–1388 the user of the lunar velocity table of John of Lignères 2 attains the final correction after the first step. However, the canon also notes that finding true syzygy after only the first step is "rare ... due to the irregularity in the lunar movement" (lines 86–89), a comment that, in light of Table 2, might prompt one to ask how well John understood the power of his method.

Table 2 also may help explain why John of Saxony selected 0;01 days as the time interval for sampling the true elongations in the second step of his method in Eq. (5). For all the lunar velocity tables compared, the absolute value of τ^* usually remains <12 min for the 1387–1388 conjunctions. Given the rate of change of the lunar velocity, a sampling interval of 0;01 days would seem to be more than adequate for computing the times of true syzygy to the nearest minute. Indeed, John could even have lengthened this sampling time without adversely degrading the precision of his method. For the cases in Table 2 with John of Lignères 2's lunar velocity table and the largest values for τ^* (18 February and 15 July 1387, 9 January and 31 August 1388), I find that I can increase the sampling interval fourfold, i.e., to 0;04 days or 96 minutes, without altering the values of τ^*, to the nearest minute, generated by Eq. (5). Hence, John's method appears to be very robust. Indeed, the method appears have been designed, given the rates of change of the lunar and solar velocities, to yield unequivocal results for the time correction to a precision of the nearest minute.

Such robustness in John's method also eliminates the second ambiguity of the canon, viz., whether the mean or true solar anomaly is to be used as the argument for the solar velocity of Eq. (2). As can be seen in Table 2, a variation of ± 1 arcsec in the solar velocity will not degrade the time correction being computed with John's two-step method.

However, the third ambiguity of John's canon, whether to employ Ptolemy's initial or final lunar model for the computation of the true elongations in the second step, can affect the outcome of the algorithms in cases where the absolute value of Δt is greater than about 12 h. The 1387–1388 conjunctions include four cases where Δt exceeds this value. Computation using the lunar velocity table of John of Lignères 2 indicates that in all four of these cases, the differences in the time correction determined using Ptolemy's final or initial lunar models in the second stop become noticeable. For 9 January and 8 February 1388, the differences are 4 min; for 2 and 31 August 1388, the differences are -5 min. For all the remaining conjunctions of 1387–1388, the proportional minutes in the second stop remain 0; i.e., the final and initial lunar models are identical.[7]

Is this third ambiguity in John of Saxony's canon thus impossible to eliminate? Not necessarily. With computer subroutines to mechanize the interpolations, one can easily calculate true elongations from the equations of the 1483 Alfonsine Tables at 1-min intervals near true syzygies. By such means, one can determine "actual" Alfonsine times of true syzygy to the nearest minute (or to any given precision). For 24 of the 25 conjunctions of 1387–1388, I find that John's method, using any of the velocity tables of Table 2 and Ptolemy's final lunar model, yields time corrections that match, to the nearest minute, the "actual" Alfonsine times of true syzygy. In one case, for 9 January 1388, the "actual" time of true syzygy is matched only by computation using the lunar velocities of John of Lignères 1; the other three lunar velocity tables yield a time correction 1 min short of the "actual" Alfonsine value. Of the several hundred other comparisons I have made (see below), Δt as determined by John of Saxony's method with the final lunar model *always* matches to the nearest minute the "actual" Alfonsine times. Given the sophistication with which John's method appears to be constructed, I am thus willing to speculate that he knew that he could match the "actual" times of true syzygy, and that he knew that Ptolemy's final lunar model was required for step two. Poulle, Chabás, and Goldstein also assumed that John intended for the final lunar model to be used in the second step of the method [Poulle, 1984, 217; Goldstein, 1992, 15; Chabás and Goldstein, 1992, 270]. If we all are correct in this assumption, then John of Saxony's method is not only robust, i.e., independent of the lunar velocity table used, but also exact, i.e., yielding time corrections that match the "actual" Alfonsine values to a precision of the nearest minute.

Late medieval users of John of Saxony's method

Another way to explore the ambiguities in John's canon would be to consider how contemporary users understood and implemented the method. Unfortunately, I know of no 14th- or 15th-century manuscripts containing true syzygy times explicitly said to be computed by John's method. Furthermore, many late medieval calendrical texts list not the time correction but the time of true syzygy silently adjusted to a local meridian and to true solar time. Unless these adjustments are described explicitly, it becomes difficult to extract time corrections from such computations. For example, in the margins of Escorial 0 II 10 a hand attributed to John of Murs computed times for the solar eclipse of 14 May 1333 and compared these with his observed times for that phenomenon [Beaujouan, 1975, 28–29]. The time of true conjunction, computed to be 2;28,44 h [after noon], is said to include the "Parisian equation of days" of 0;20,54 h (John of Murs noted that "according to John of Lignères," this latter value is 0;21,16 h). When using "tables of mean motions" (rather than presumably computing the mean motions), John of Murs found the time of true conjunction to be 2;27,23 h [after noon]. In his analysis of these marginalia, G. Beaujouan indicated that in another computation John of Murs had given the longitude difference between Toledo (the meridian of the Alfonsine Tables) and Paris as 0;48 min of time.[8] From this information, John of Murs's time correction can be extracted. According to the 1483 Alfonsine mean motions, the mean syzygy on 14 May 1333 occurred at 8;50 h after noon, for the meridian of Toledo.

[7] For 1389, John's method yields differences between the final and simple lunar models for 6 of the 12 conjunctions; in 1390, for 5 of the 12 conjunctions.

[8] According to Kremer and Dobrzycki [1998, 196], 48 min of time was the standard Toledo–Paris distance for users of the Alfonsine Tables in manuscript.

Subtracting the equation of days and longitude adjustment from John of Murs's time of true syzygy yields a time correction of $-7;30$ h. For this date, John of Saxony's method yields a time correction of $-7;31$ h. For a correction of this magnitude, however, the proportional parts remain zero in the second step of John of Saxony's method, so that one cannot determine whether John of Murs, if he employed that method, used Ptolemy's final or initial lunar model. For such a determination, longer series of syzygy computations are required.

The earliest extant lists (known to me) of computed times for eclipses or syzygies, extending over several Metonic cycles, are to be found in English manuscripts prepared by calendar-makers related to Oxford [Thorndike, 1957; North, 1988, 87–101]. Working before John of Saxony composed his canon and probably before the Oxford versions of the Alfonsine Tables were available, Walter of Odington and Walter of Elveden (connected to Cambridge) computed syzygy times for cycles from 1292 to 1367 and 1330 to 1386, respectively.[9] In the 1380s, John Somer and Nicholas of Lynn computed times of syzygies and eclipses for cycles from 1387 to 1462. Extant in dozens of manuscripts and recently edited, the *Kalendaria* of Somer and Nicholas provide syzygy times to the nearest minute, which enables exploration of their possible sources. Although the syzygy times in these two texts are quite close, they are not identical. As noted by J.D. North, it seems reasonable to assume that Somer and Nicholas worked independently, probably using slightly different equations of time [North, 1988, 100].

In their canons, both Somer and Nicholas name Oxford as their meridian, but they do not specify a longitudinal distance between that town and Toledo.[10] The canons also do not indicate whether the syzygy times of the calendars are for mean or true solar time (i.e., adjusted by addition of the equation of time). A comparison of those times, however, with times derived by John of Saxony's method and from Georg Peurbach's double-entry table [Tannstetter, 1514] suggests that both English calendars were computed for a longitude 16 time minutes east of Toledo and that an equation of time very close to that printed in the 1483 Alfonsine Tables was also applied.

Table 3 presents the time corrections by John of Saxony's method with the John of Lignères 2 lunar velocity table and by Peurbach's table, for the conjunctions of the first three years covered by Somer's and Nicholas's calendars. For the 10 cases in which the outcome of John of Saxony's method depends on whether Ptolemy's final or initial lunar model is used in step two, I list both values in col. 2 (final/initial). John of Saxony's method, with Ptolemy's final lunar model, produces time corrections that correspond, to the nearest minute, to the "actual" Alfonsine times of true conjunction (except for 9 January 1388). As can be seen from cols. 2–3, time corrections of John of Saxony (initial lunar model) and Peurbach generally differ by no more than ± 1 min (except for 19 September 1389). Peurbach's table clearly implements an algorithm based on Ptolemy's initial rather than final lunar model.[11]

9 North [1988, 93] suggests that Walter of Elveden used the Toledan Tables.

10 The modern editors of these two calendars do not discuss how John or Nicholas might have computed the times of syzygy; nor do they investigate the meridians used in such computations. Eisner [1980, 4], notes that scholars at Merton College had "established" a longitudinal distance of "16 min" between Oxford and Toledo, but Eisner did not realize that this figure refers to minutes of time. North [1988, 95] agrees that Oxford tables generally use 4° [of longitude] as the "standard figure" for the Oxford–Toledo distance.

11 Peurbach's table, first printed in Tanstetter [1514, sigs. a3v–d3r], is an expanded version (reduced intervals between the arguments) of a similar table attributed to John of Gmunden, which in turn is identical to one composed by John of Murs c. 1330 (see Porres and Chabás [2001, 64], Chabás and Goldstein [1997, 100–101]). The double-entry table expanded by Peurbach thus originated at roughly the same time as did John of Saxony's canon.

Table 3
Sources for conjunctions in the English calendars[a]

Date	Time correction in h		John Somer[b]		Nicholas of Lynn	
1	2	3	4	5	6	7
1387–1389	John Saxony	Peurb.	JS-Cal.	P-Cal.	JS-Cal.	P-Cal.
20.Jan	8;02	8;03	0	1	−1	0
18.Feb	5;15	5;16	−1	0	−1	0
19.Mar	1;16	1;17	0	1	0	1
18.Apr	−3;05	−3;05	0	0	0	0
17.May	−7;04	−7;04	0	0	0	0
15.Jun	−9;53	−9;53	0	0	1	1
15.Jul	−10;56	−10;57	−1	−2	−1	−2
13.Aug	−9;49	−9;50	0	−1	0	−1
12.Sep	−6;29	−6;29	0	0	0	0
12.Oct	−1;29	−1;29	0	0	−2	−2
11.Nov	4;07	4;07	0	0	0	0
10.Dec	8;55	8;55	0	0	−1	−1
09.Jan	11;43/11;39	11;38	0/−4	−6	2/−2	−4
08.Feb	11;49/11;45	11;45	0/−4	−4	3/−1	−1
08.Mar	9;16	9;17	0	1	−1	0
06.Apr	4;54	4;54	0	0	−1	−1
06.May	−0;29	−0;30	0	−1	−1	−2
04.Jun	−5;53	−5;54	−1	−2	0	−1
03.Jul	−10;18	−10;18	−1	−1	1	1
02.Aug	−12;59/−12;54	−12;54	−1/4	4	−1/4	4
31.Aug	−13;00/−12;55	−12;56	−1/4	3	−3/2	1
30.Sep	−10;07	−10;07	−1	−1	0	0
30.Oct	−4;48	−4;47	−1	0	−2	−1
28.Nov	1;51	1;52	−1	0	−2	−1
28.Dec	8;09	8;09	0	0	−1	−1
27.Jan	12;29/12;24	12;24	0/−5	−5	0/−5	−5
26.Feb	13;49/13;44	13;45	−1/−6	−5	−1/−6	−5
27.Mar	12;07/12;03	12;03	−1/−5	−5	0/−4	−4
25.Apr	7;58	7;58	−1	−1	−2	−2
25.May	2;25	2;25	0	0	−1	−1
23.Jun	−3;35	−3;35	−1	−1	−1	−1
22.Jul	−8;58	−8;58	−1	−1	−1	−1
21.Aug	−12;49/−12;44	−12;45	−1/4	3	−1/4	3
19.Sep	−14;03/−13;57	−13;59	−1/4	3	0/5	4
19.Oct	−12;13/−12;08	−12;08	−1/4	4	−2/3	3
17.Nov	−7;22	−7;23	0	−1	0	−1
17.Dec	−0;45	−0;46	1	0	−1	−2

[a] Eisner [1980, 66–132], Mooney [1998, 117–39]. [b] Cols. 4–7 list minutes of time. Cols. 4 and 6 are computed as follows: (t of mean syzygy, as per 1483 Alfonsine Tables) + (John of Saxony's Δt) − (time of true syzygy as given in the respective calendars) + 0;16 + equation of time. Cols. 5 and 7 take Δt from Peurbach's double-entry table. I apply the equation of time from the 1483 Alfonsine Tables (maximum of 7;54° or 0;31,36 h from Sco 8–9). Somer's and Nicholas's canons and calendars offer no hints concerning sources for their equations of time.

Table 4
Sources for true syzygies in Regiomontanus's Almanac[a]

Date	Time correction in h		Differences in min	
1 1448, 1453–1454	2 John of Saxony	3 Peurbach	4 JS − Regio	5 P − Regio
05.Jan.48	3;15	3;16	0	1
20.Jan.48	3;27	3;27	0	0
04.Feb.48	0;35	0;35	0	0
19.Feb.48	9;08	9;08	10[b]	10
04.Mar.48	−2;22	−2;22	0	0
20.Mar.48	12;34/12;29	12;30	0/−5	−4
03.Apr.48	−5;04	−5;03	0	1
18.Apr.48	13;08/13;03	13;04	0/−5	−3
02.May.48	−7;01	−7;00	0	1
18.May.48	10;59/10;58	10;58	1/0	0
01.Jun.48	−7;47	−7;47	−1	−1
16.Jun.48	6;53	6;54	−1	0
30.Jun.48	−7;09	−7;09	−1	−1
15.Jul.48	1;39	1;39	0	0
30.Jul.48	−5;04	−5;05	0	−1
09.Jan.53	−2;43	−2;43	0	0
24.Jan.53	5;23	5;23	−1	−1
08.Feb.53	3;24	3;24	1	1
23.Feb.53	2;11	2;12	0	1
10.Mar.53	8;27	8;26	1	0
24.Mar.53	−1;37	−1;36	0	1
08.Apr.53	11;27/11;24	11;24	2/−1	−1
22.Apr.53	−5;17	−5;18	1	0
08.May.53	11;56/11;51	11;51	5/0	0
22.May.53	−8;10	−8;10	0	0
06.Jun.53	9;58	9;58	1	1
20.Jun.53	−9;38	−9;39	0	−1
06.Jul.53	6;20	6;19	0	−1
20.Jul.53	−9;19	−9;20	0	−1
04.Aug.53	1;40	1;39	1	0
14.Jan.54	11;04	11;04	1	1
28.Jan.54	−4;40	−4;39	0	1
12.Feb.54	9;43	9;44	0	1
27.Feb.54	−0;09	−0;09	0	0
14.Mar.54	6;18	6;18	0	0
28.Mar.54	4;07	4;07	0	0
12.Apr.54	1;34	1;34	0	0
27.Apr.54	7;15	7;16	0	1
11.May.54	−3;35	−3;35	0	0
27.May.54	8;46	8;48	0	2
10.Jun.54	−8;14	−8;13	0	1
25.Jun.54	8;35	8;36	0	1
09.Jul.54	−11;31/−11;27	−11;28	−1/3	2
25.Jul.54	6;50	6;49	0	−1
08.Aug.54	−12;39/−12;34	−12;35	−2/3	2

Table 4 (*Continued*)

Date	Time correction in h		Differences in min	
1	2	3	4	5
1448, 1453–1454	John of Saxony	Peurbach	JS − Regio	P − Regio
23.Aug.54	3;59	3;58	0	−1
06.Sep.54	−11;04/−11;03	−11;04	0/1	0
21.Sep.54	0;35	0;35	0	0
06.Oct.54	−7;00	−7;00	0	0
21.Oct.54	−2;44	−2;44	0	0
05.Nov.54	−1;04	−1;04	0	0
19.Nov.54	−5;21	−5;22	0	−1
04.Dec.54	5;17	5;18	−1	0
19.Dec.54	−6;49	−6;49	0	0

[a] Vienna, ÖNB cvp 4988, ff. 8r–12v, 29v–34v, 47v–56v. [b] Presumably Regiomontanus erroneously recorded one digit in this time. Such large errors are extremely rare in his ephemerides and horoscopes.

Somer's calendar appears to be based on Ptolemy's final lunar model. As can be seen in col. 4, Somer's times so smoothly follow John of Saxony's that one might conclude that Somer (or his source) had scrupulously employed John of Saxony's method. I know of no other procedures by which a 14th-century astronomer could have so closely approximated the "actual" Alfonsine times of true conjunction. The case is not so clear for Nicholas's calendar. For the large time corrections in 1389, Nicholas appears to have followed Ptolemy's final lunar model; for 1388, the data appear inconclusive. Either Nicholas employed an equation of time that differs somewhat sporadically from that of the 1483 Alfonsine Tables, or he was a less precise calculator. In any case, the data of cols. 6–7 do not indicate unambiguously which technique Nicholas may have used to determine his time corrections.

For a final example, I turn to Regiomontanus, undoubtedly the most prolific computer of ephemerides in the 15th century. As noted above, Regiomontanus accompanied his annual ephemerides with horoscopes for each syzygy, listing on the charts the times of true syzygy in both mean and true (i.e., with equation of time added) solar time. Unlike those of Somer and Nicholas, Regiomontanus's computational procedures thus can be investigated without obscuration by an unknown equation of time.

Table 4 lists comparisons for the first three years of the charts accompanying Regiomontanus's early unpublished almanac. Unlike John Somer, the young Regiomontanus appears to have varied his computational procedures year by year (although for the years from 1448 through 1462 he consistently used a local meridian of 80 time minutes east of Toledo). For 1448, Regiomontanus's computed times of true syzygy match those of John of Saxony's method with Ptolemy's final lunar model to ±1 min. For 1453, the next set of charts in his almanac, Regiomontanus appears to have used Ptolemy's initial lunar model, as his syzygy times match those of Peurbach's double-entry table to ±1 min. For 1454, he returned to Ptolemy's final lunar model and John of Saxony's method. A continuation of this analysis for the remaining syzygy times in his unpublished almanac finds that Regiomontanus used John of Saxony's method with the final lunar model for only one more year (1456). For the other years in his unpublished almanac (1455, 1457, 1458–1459, 1459–1460, 1461–1462), Regiomontanus appears to have used Ptolemy's initial lunar model or Peurbach's double-entry table.[12] For all of these several hundred true syzygy corrections, Regiomontanus's interpolations or computations match my results from Peurbach's table or John of Saxony's method to ± 1 min or better.

These comparisons cannot prove that Regiomontanus employed Peurbach's double-entry tables. But for the years of 1448, 1454, and 1456, he probably followed John of Saxony's method with Ptolemy's final lunar model, since no other known procedure could have yielded time corrections so close to the "actual" Alfonsine values. In Regiomontanus and John of Somer, John of Saxony's method appears to have found two highly competent users. Despite its computational complexity, the method produced strikingly precise Alfonsine results for those two astronomers.

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[12] Regiomontanus's autograph of Peurbach's eclipse tables, a work not printed until 1514, is extant in Nuremberg, Stadtbibliothek Cent V 57, ff. 10r–19v, 108r–53v (see Neske [1997, 90–91]). The times of true syzygy for 1475–1506 that appear in Regiomontanus's massive printed ephemerides also are derived from Ptolemy's initial lunar model or Peurbach's double-entry table.

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